Lecture 2.

Determinants. The basic properties of determinants.

Minor and the algebraic complement set.

Let A be a square matrix, and write r_i for the i-th row.

Definition: The determinant is a real-valued function of the corresponding matrix. So, it is a number – not a matrix.

The determinant of matrix A is denoted by det A or $|A|$.

The determinant of the 1×1 matrix $A = [a]$ is equal to a.

Let's look at the case where A is a n × n matrix. Then its determinant may be calculated as

$$\det A = a_{i1} \cdot A_{i1} + \cdots + a_{in} \cdot A_{in} \text{ for any row } i$$

and the same value of the determinant is obtained using a similar evaluation by any column. In the last formula A_{ij} cofactor to element a_{ij} which consists of the complement minor of element a_{ij} with the sign which is defined as $(-1)^{i+j}$.

We can write determinant of A as $\det(A) = \det(r_1, r_2, \ldots, r_n)$. It is completely determined by the following four properties:

1. Multiplying a row by the constant c multiplies the determinant by c: $\det(r_1, r_2, \ldots, cr_i, \ldots, r_n) = c \det(r_1, r_2, \ldots, r_n)$
2. If row i is the sum of the two row vectors x and y, then the determinant is the sum of the two corresponding determinants: $\det(r_1, r_2 \ldots, x+y, \ldots, r_n) = \det(r_1, r_2 \ldots, x, \ldots, r_n) + \det(r_1, r_2 \ldots, y, \ldots, r_n)$
3. Interchanging any two rows of the matrix changes the sign of the determinant: $\det(\ldots, ri, \ldots, rj \ldots) = -\det(\ldots, rj, \ldots, ri, \ldots)$
4. The determinant of any identity matrix is 1.
5. The determinant of A is the same as that of its transpose A^t.
6. If A and B are square matrices of the same size, then det(AB) = det(A) det(B)